

MARKER CODE				



STUDENT ENROLMENT NUMBER (SEN)									

TONGA NATIONAL FORM SEVEN CERTIFICATE 2022

MATHEMATICS with STATISTICS

QUESTION AND ANSWER BOOKLET

Time allowed: 3 Hours

INSTRUCTIONS:

1. Write your **Student Enrolment Number (SEN)** on the top right-hand corner of this page.
2. This paper consists of **SIX SECTIONS** and is out of 80 weighted scores.

SECTION	STRANDS	TOTAL SKILL LEVEL
ONE	PROBABILITY	20
TWO	ALGEBRA	17
THREE	SEQUENCE AND SERIES	9
FOUR	COMBINATIONS AND BINOMIAL THEOREM	14
FIVE	CALCULUS	10
SIX	TIME SERIES	10
TOTAL		80

3. Answer ALL QUESTIONS. Write your answers in the spaces provided in this booklet.
4. Use a **BLUE** or **BLACK** ball point pen only for writing. Use a pencil for drawing if required.
5. If you need more space for answers, ask the supervisor for extra paper. Write your **Student Enrolment Number (SEN)** on each additional sheet, number the questions clearly and insert them in the appropriate places in this booklet.

- NOTE:**
- i. There should be a **Mathematics Formulae and Tables Booklet (No. 503/2)** attached to this booklet.
 - ii. Non-programmable calculators **only** are allowed into the examination room.
 - iii. Unless stated, diagrams are not drawn to scale.
6. Check that this booklet contain pages **2-19** in the correct order and that page 19 has been deliberately left blank.

YOU MUST HAND IN THIS BOOKLET TO THE SUPERVISOR BEFORE YOU LEAVE THE EXAMINATION ROOM.

SECTION ONE:

PROBABILITY

1. The following probability distribution table tells us the probability that the TCC's soccer team scores a certain number of goals in their first game:

Goal (x)	0	1	2	3	4
Probability $P(X = x)$	0.18	$2k$	0.35	0.11	0.02

- a. Find the value of k .

Skill level 2	
2	
1	
0	
NR	

- b. Calculate the probability that the TCC team scores at most **TWO** (2) goals in their first game.

Skill level 2	
2	
1	
0	
NR	

2. The table below shows the Stat results Test of students at Moli High School.

	Pass	Fail
Boys	100	50
Girls	75	25

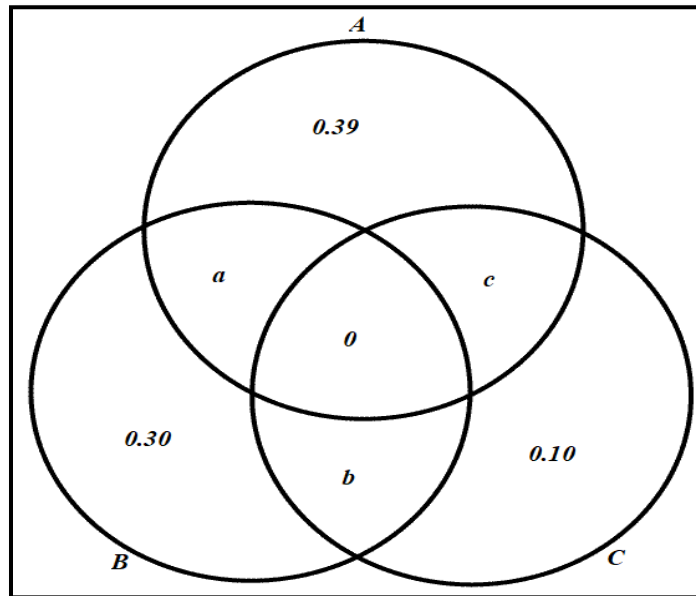
If a student is selected at random, calculate the probability that the student is a girl, given that she passed the Test.

Skill level 1	
1	
0	
NR	

3. The X has a normal distribution with a mean of 71.3. Given that $P(X < 80) = 0.75$, calculate the *standard deviation*.

Skill level 3	
3	
2	
1	
0	
NR	

4. The Venn diagram shows events A , B and C .
 Events A and B are **mutually exclusive**.
 Events A and C are **independent**.



- a. Find the value of a .

Skill level 1	
1	
0	
NR	

- b. Calculate the values of ***b*** and ***c***.

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Skill level 4	
4	
3	
2	
1	
0	
NR	

5. Sunshine Rental has several vehicles available for hire. The probability that a vehicle breaks down when it is rented out is 0.3.

One day, 10 vehicles were rented. Calculate the ***variance*** of vehicles that would break down.

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Skill level 1	
1	
0	
NR	

6. A 5L bucket of water is taken from a tank. The water contains 35 mosquito larvae. A 1L mug of water is taken from the bucket for further analysis.

The number of mosquito larvae can be modelled by a Poisson Distribution.

Calculate the probability that the mug contains less than 2 larvae.

Skill level 3	
3	
2	
1	
0	
NR	

7. A student is chosen at random from the Seventh Form population. The probability that the student is a boy is 0.45. The probability that a boy takes Statistics is 0.15, while the probability that a girl takes Statistics is 0.2.

Calculate the probability that a student chosen at random is a girl who studies Statistics.

(Hint: Draw a Probability Tree)

Skill level 3	
3	
2	
1	
0	
NR	

SECTION TWO:**ALGEBRA**

1. A 2×2 system of linear equations has a *unique solution*.
State in *geometrical terms* the meaning of unique solution.

Skill level 1	
1	
0	
NR	

2. Solve this system of **3 x 3** simultaneous equations.

$$4x - y + z = 8$$

$$3x + 2y - 3z = 47$$

$$x - 3y + 2z = -23$$

Skill level 3	
3	
2	
1	
0	
NR	

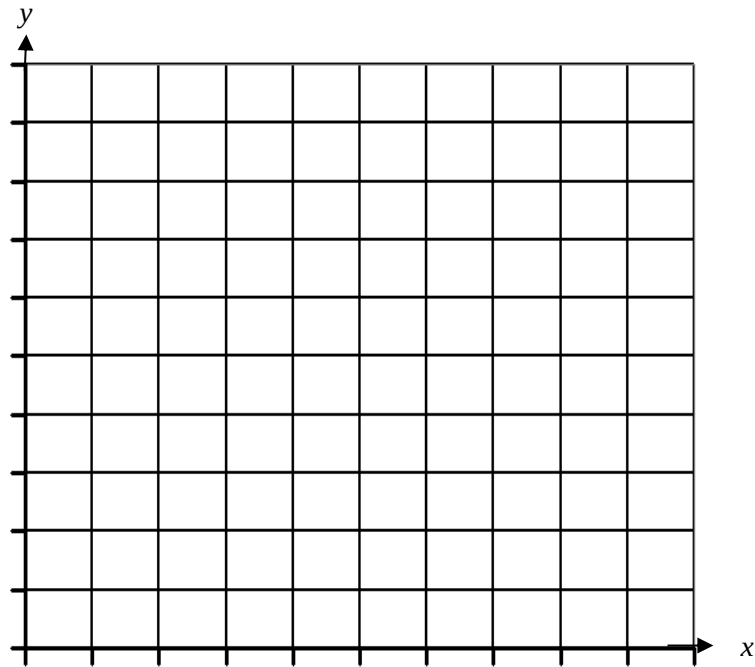
3. Solve $3^{2x-1} = 243$.

Skill level 2	
2	
1	
0	
NR	

4. Sketch the graph of $y = x^{-1}$.

Skill level 2	
2	
1	
0	
NR	

5. a. Sketch the linear graphs of these inequalities and indicate the feasible region.
 $y \geq 0$, $x \geq 0$, $x + y \leq 5$, $x + 2y \leq 8$ (use the grid given below)



Skill level 3	
3	
2	
1	
0	
NR	

- b. The expression $3x + 5y$ is to be maximized subject to the inequalities in a.
 Study the table given below and answer the following question.

Vertex	Value of $3x + 5y$
(0,0)	0
(0,4)	20
(2, 3)	21
(5,0)	15

State the maximum value of $3x + 5y$.

Skill level 1	
1	
0	
NR	

6. The table below shows the result for 20 iterations for both Bisection and Newton-Raphson methods in finding the positive root of $x^4 - 3 = 0$. The starting interval for the Bisection is $[1, 2]$. Use the table to answer the following questions.

Iteration Number	Bisection Method		Newton-Raphson Method
0	1.000000	2.000000	1.500000
1	1.000000	1.500000	1.347222
2	1.250000	1.500000	1.317138
3	1.250000	1.375000	1.316075
4	1.312500	1.375000	1.316074
5	1.312500	1.347500	1.316074
6	1.312500	1.343750	1.316074
7	1.312500	1.328125	1.316074
8	1.312500	1.320313	1.316074
9	1.312500	1.316407	1.316074
10	1.314454	1.316407	1.316074
11	1.315431	1.316407	1.316074
12	1.315919	1.316407	1.316074
13	1.315919	1.316163	1.316074
14	1.316041	1.316163	1.316074
15	1.316041	1.316102	1.316074
16	1.316072	1.316102	1.316074
17	1.316072	1.316087	1.316074
18	1.316072	1.316079	1.316074
19	1.316072	1.316075	1.316074
20	1.316073	1.316075	1.316074

- a. State the starting value for the Newton-Raphson method.

Skill level 1	
1	
0	
NR	

- b. State the value for the positive root to 3 decimal places. Then make a statement to justify why Newton-Raphson method is a better method to improve the approximation to a given precision.

[illegible]

Skill level 4	
4	
3	
2	
1	
0	
NR	

SECTION THREE:

SEQUENCE AND SERIES

1. Evaluate $\sum_{n=1}^{\infty} \frac{2}{3^n}$

Skill level 2	
2	
1	
0	
NR	

2. The sum of the first n terms of an Arithmetic sequence is $S_n = n(6 - n)$. Find the *common difference*, d .

Skill level 3	
3	
2	
1	
0	
NR	

3. Write down the first three terms for the series $(1-x)e^{2x}$.

Skill level 2	
2	
1	
0	
NR	

4. The sequence $\langle 4, 1, \frac{1}{4}, \dots \rangle$ is a geometric. Find T_{10}

Skill level 2	
2	
1	
0	
NR	

SECTION FOUR: COMBINATIONS AND BINOMIAL THEOREM

1. Simplify $\frac{3r(r-1)!}{12r!}$

Skill level 2	
2	
1	
0	
NR	

2. Seven (7) students line up to be photographed. Three (3) of them are Prefects. Calculate the number of possible ways of lining them up, if the Prefects must stand together.

Skill level 3	
3	
2	
1	
0	
NR	

3. The Ministry of Education and Training has 25 qualified teachers, 15 to be assigned to Vava'u and 10 to Ha'apai. Five of the teachers refuse to go to Ha'apai.

Find the number of ways the 25 teachers can be assigned.

Skill level 2	
2	
1	
0	
NR	

4. Solve ${}^nC_1 = {}^nP_2$

Skill level 3	
3	
2	
1	
0	
NR	

5. Use ${}^nC_r = \frac{n!}{r!(n-r)!}$ to prove that ${}^nC_{r-1} + {}^nC_{r-2} = {}^{n+1}C_{r-1}$

Skill level 4	
4	
3	
2	
1	
0	
NR	

SECTION FIVE:

CALCULUS

1. Differentiate $\frac{1}{3x}$.

Skill level 1	
1	
0	
NR	

2. Determine the nature of the turning points for the curve $y = x^3 - 3x - 1$.

Skill level 3	
3	
2	
1	
0	
NR	

3. A population growth is modelled by the differential equation

$$\frac{dP}{dt} = kP$$

where P is the population, t is the time measured in days and k is a constant.

- a. Show by differentiation that the solution to the differential equation is $P = P_0 e^{kt}$

Skill level 2	
2	
1	
0	
NR	

- b. At the beginning it was estimated that the population is 200. Thirty days later the population increases to 350. Estimate the *population size* after ninety (90) days.

Hint: First find the values of P_0 and k .

Skill level 4	
4	
3	
2	
1	
0	
NR	

SECTION SIX:**TIME SERIES**

1. The time series in the table below gives the numbers of registered unemployed in a country from 1992 to 1996.

Time		Number	Moving Mean (order 3)	Individual seasonal effects
1992	31-Jul	3210		
	30-Nov	2843	2940	-97
1993	31-Mar	2767	2877	-110
	31-Jul	3021	2800	221
	30-Nov	2612	2726	-114
1994	31-Mar	2545	2684	-139
	31-Jul	2895	2611	284
	30-Nov	2393	2534	-141
1995	31-Mar	2314	A	**
	31-Jul	2631	2371	260
	30-Nov	2168	2292	-124
1996	31-Mar	2077	2213	-136
	31-Jul	2394		

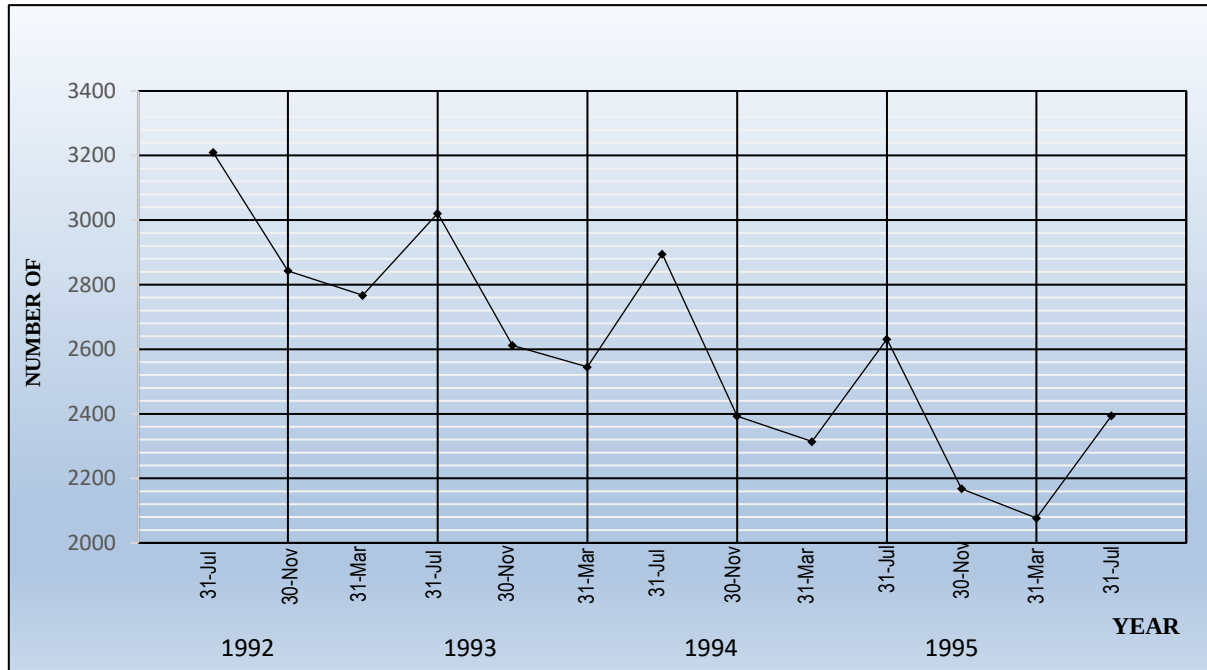
- a. Find the value of **A**.

Skill level 1	
1	
0	
NR	

- b. Describe the reason(s) for smoothing this data by order 3.

Skill level 2	
2	
1	
0	
NR	

- c. The graph below shows the number of registered unemployed for 5 years.



- i. Seasonal variation is one of the time series components shown by the graph above. Name another component.

Skill level 1	
1	
0	
NR	

- ii. Describe the *Seasonal variation* with reference to the number of registered unemployed.

Skill level 2	
2	
1	
0	
NR	

- iii. Draw the trend line (moving mean) into the graph in c and predict the unemployment levels for 30th Nov, 1966.



Skill level 4	
4	
3	
2	
1	
0	
NR	

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